Superconducting pairing of electrons coupled to transverse gauge field

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Suk Bum Chung, Ipsita Mandal, Sri Raghu, S. C.
PRB 045127 (2013)

Z. Wang, Ipsita Mandal, Suk Bum Chung, S. C.
Annals of Physics, 351, 727 (2014)

Z. Wang and S. C. PRB 94, 165138 (2016)

Nicholas Rombes and S. C., In preparation.

Higher angular momentum pairing from transverse gauge interactions

Pairing in half-filled Landau level, HLR theory

Particle-hole symmetric composite fermions in half-filled Landau level

Specific heat and pairing of Dirac composite fermions in the half-filled Landau level

- Fermions coupled to emergent gauge fields—spin liquids, normal state of cuprates, half-filled Landau level, etc.

- A well defined model for a non-fermi liquid

- Attraction from repulsion—unlike Kohn and Luttinger, superconductivity takes place at a finite coupling— dynamic screening and the attraction is in the frequency domain

- Quantum phase transition between a composite Fermi liquid to a superconductor

- Comparison between HLR theory and Son theory $\nu = 1/2$

Non relativistic fermions coupled to a transverse U(1) gauge field

Holstein, Norton and Pincus, Phys. Rev. B 8, 2649 (1979)

- A beautiful example of a non-fermi liquid
- Specific heat in 3D: $C \propto T \ln T$
- A renormalization group analysis in 2D: Chakravarty, Norton, Syljuasen Phys. Rev. Lett. 74, 1423 (1995), also shows non-fermi liquid behavior

Superconductivity ?

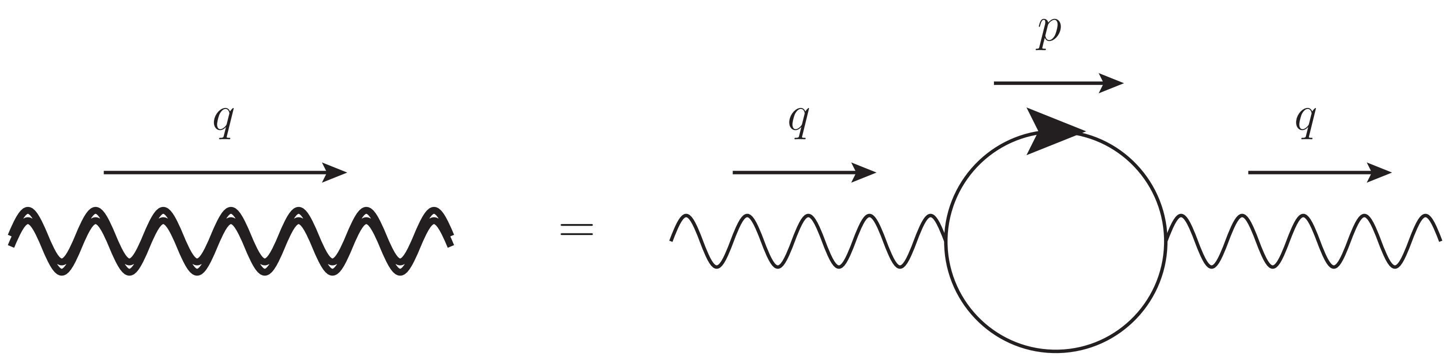
$$S_F = \int \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} \psi^\dagger(\mathbf{k},\omega)\left[i\omega - (\varepsilon_k - \mu)\right]\psi(\mathbf{k},\omega)$$

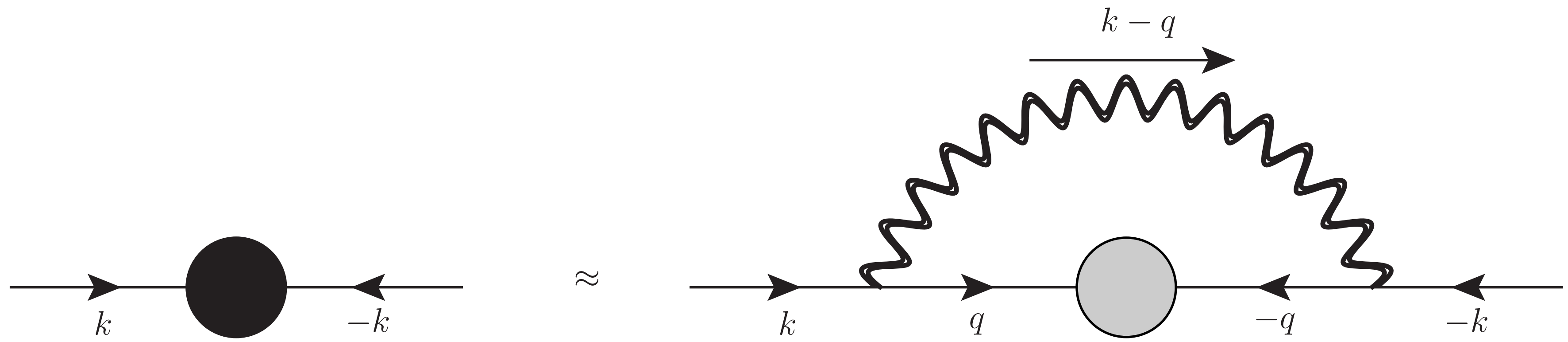
$$S_G = \int \frac{d\nu}{2\pi} \int \frac{d^3q}{(2\pi)^3} A_i^\dagger(\mathbf{q},\nu)\left[q^2+\nu^2\right]\left(\delta_{ij} - q_i q_j/q^2\right) A_j(\mathbf{q},\nu)$$

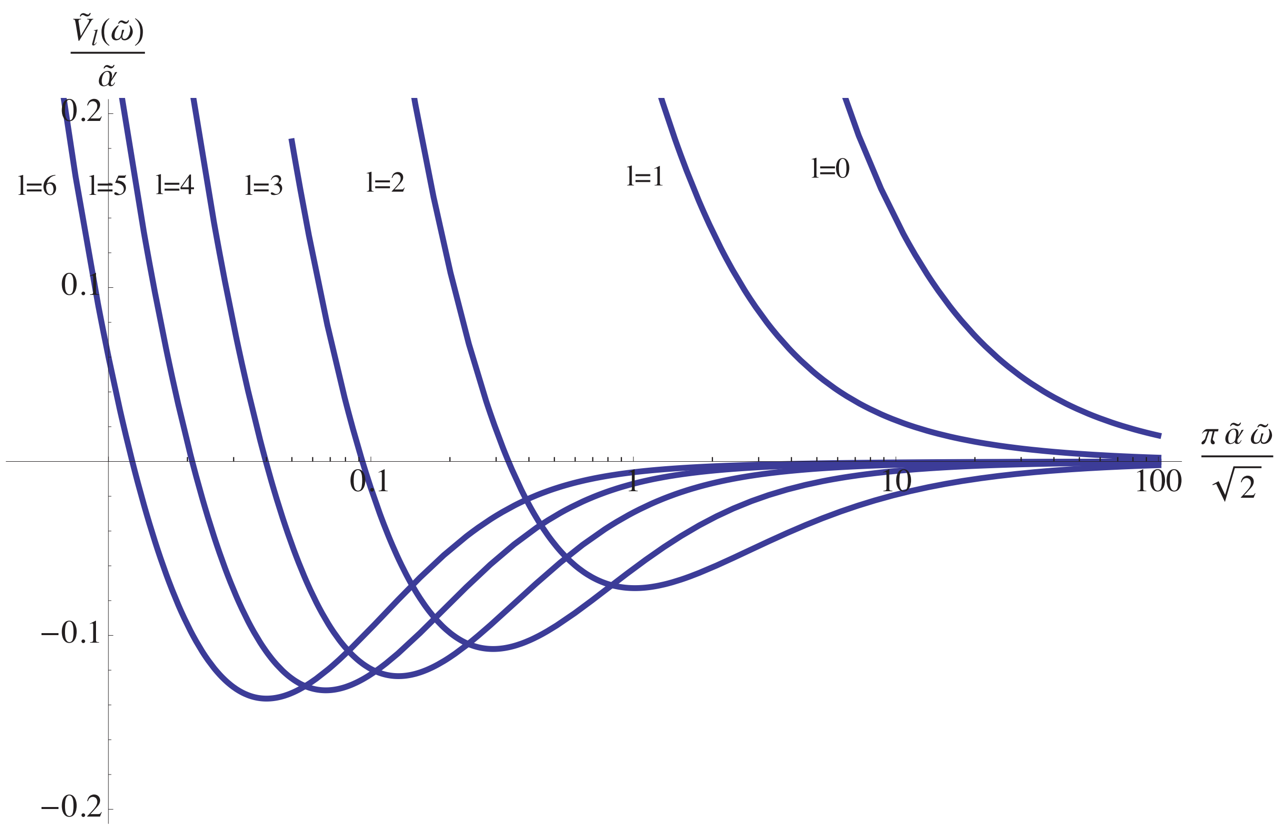
$$S_{\rm int} = \frac{g}{m}\int \frac{d\omega}{2\pi}\int \frac{d\nu}{2\pi}\int \frac{d^3p}{(2\pi)^3}\int \frac{d^3k}{(2\pi)^3}\psi^\dagger(\mathbf{k},\nu)\{A_i(\mathbf{k}-\mathbf{p},\nu-\omega)p_i\}\,\psi(\mathbf{p},\omega)$$

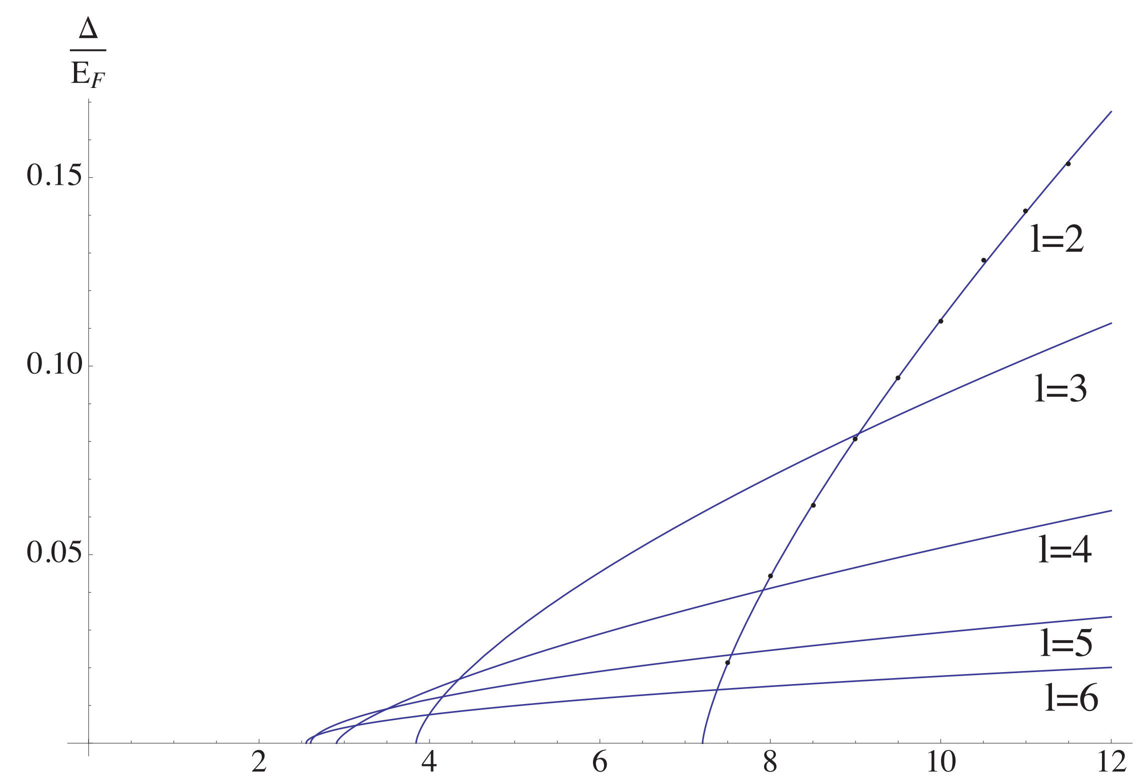
$$S_G = \int \frac{d\nu}{2\pi} \int \frac{d^3q}{(2\pi)^3} A_i^\dagger(\mathbf{q},\nu)\left[q^2+\nu^2+\gamma|\nu|/qv_F\right]\left(\delta_{ij} - q_i q_j/q^2\right) A_j(\mathbf{q},\nu),$$

$$D_{ij}(\mathbf{k},\nu) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\frac{1}{k^2+\gamma\frac{|\nu|}{kv_F}}$$









Similar results in 2D

Half-Filled Landau level—Son Composite fermion theory

$$S_{\rm CF} = \int d\tau d^2x \{\bar{\psi}\gamma_\mu(\partial_\mu + i a_\mu)\psi - i\frac{B}{4\pi}a_0\}$$

$$S_{\rm int} = \frac{1}{2}\int \frac{d\Omega d^2\vec{q}}{(2\pi)^3} J_{\rm T}(\Omega,\vec{q})\mathcal{D}_{\rm T}^{(RPA)}(\Omega,\vec{q})J_{\rm T}(-\Omega,-\vec{q})$$

$$J_i(\Omega,\vec{q}) = v_F\int d\omega d^2\vec{k}/(2\pi)^3\psi^\dagger(\omega+\Omega,\vec{k}+\vec{q})\, i\gamma_0\gamma_i\psi(\omega,\vec{k})$$

$$J_{\rm T}(\Omega,\vec{q}) = \epsilon_{ij}\hat{q}_i J_j(\Omega,\vec{q})$$

$$P_{\vec{k}}^{(+)}\psi(\omega,\vec{k}) = \frac{1}{\sqrt{2}}\begin{pmatrix} ie^{-i\theta_{\vec{k}}} \\ 1 \end{pmatrix}\chi(\omega,\vec{k})$$

$$P_{\vec{k}}^{(+)} \equiv \frac{1}{2}[1 + i\gamma_0\boldsymbol{\gamma}\cdot\hat{\vec{k}}]$$

Kachru, Mulligan, Torroba, Wang PRB 92, 235105 (2015)

Projection operator is necessary. Otherwise one will mix particles near the CF Fermi surface with their antiparticles buried deep in the Dirac sea. χ is a scalar field at the Fermi surface

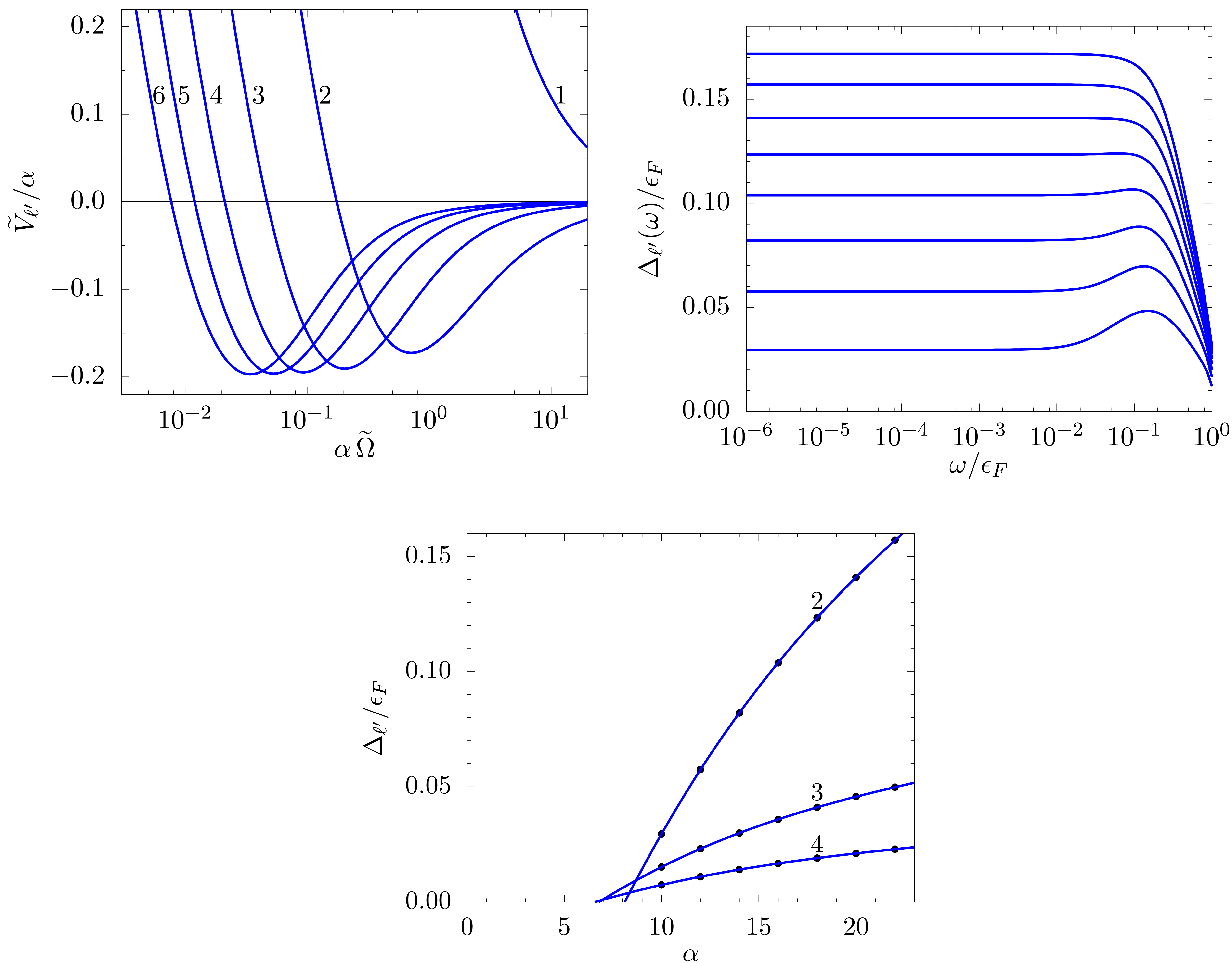
$$V_{\ell'}(i\Omega) = \alpha \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} \frac{e^{i(\ell'-1)\theta}}{|\sin\frac{\theta}{2}|} \frac{2}{1+\alpha\frac{|\Omega|}{\sin^2\frac{\theta}{2}}}.$$

We are ultimately interested in the pairing of $P^{(+)}_{\vec{k}}\psi(k)$ for which we use angular momentum ℓ, while for $\chi(k)$ we use ℓ'

The relation between ℓ and ℓ' depends on whether or not pairing is in the singlet or triplet channel

$$\alpha = \frac{v_F k_F}{e^2 k_F/\epsilon_r}$$

The ratio between kinetic to potential energy drives the quantum phase transition



0.2
0.1
0.0
−0.1
−0.2
$\widetilde{V}_{\ell'}/\alpha$
6 5 4 3 2 1
10^{-2} 10^{-1} 10^{0} 10^{1}
$\alpha\,\widetilde{\Omega}$
0.15
0.10
0.05
0.00
$\Delta_{\ell'}(\omega)/\epsilon_F$
10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0}
ω/ϵ_F
0.15
0.10
0.05
0.00
$\Delta_{\ell'}/\epsilon_F$
2
3
4
0 5 10 15 20
α

Order Parameter Symmetry

$$\hat{\Delta} = [\Delta_s(k) + \mathbf{d}(\mathbf{k}).\boldsymbol{\sigma}] i\sigma_2$$

The singlet and triplet channels are approximately decoupled for low energies

singlet: $$\hat{\Delta}(k) = \langle \psi^T(-k) P^+_{-\mathbf{k}} i\sigma_2 P^+_{\mathbf{k}} \psi(k) \rangle$$

$$\hat{\Delta}(k) \propto i\sigma_2 e^{-i\theta_k} \langle \chi(k)\chi(-k) \rangle$$

If $\langle \chi(k)\chi(-k) \rangle$ **is in** ℓ' **channel** $\hat{\Delta}(k)$ **is in** $\ell = \ell' - 1$

Son

Moore-Read Pfaffian : $\ell = -2$

Moore read anti-pfaffian : $\ell = 2$

HLR

Pfaffian : $\ell = -1$

anti-pfaffian : $\ell = 3$

Half-filled Landau level—Halperin, Lee, Read theory

Pairing of composite fermions in half-filled Landau level state is reexamined by solving the BCS gap equation with full frequency dependent effective interactions. Our results show that there can be a continuous transition from the Halperin-Lee-Read state to a chiral odd ℓ Cooper pairing state for both short-range contact interaction and long-range Coulomb-like interaction, contradicting the conclusion of a first order pairing transition for short range interaction obtained by Bonesteel [Phys. Rev. Lett. {\bf 82}, 984 (1999)], in which the full frequency dependence of the effective interaction was not taken into account. We construct the phase diagrams for both short and long range interaction cases and show that they can be quite different from each other for angular momentum channels $\ell \geq 3$. Remarkably, full frequency dependent analysis applied to the bilayer Hall system with a total filling fraction $\nu = 1/2 + 1/2$ does not change the qualitative conclusions of Bonesteel, McDonald and Nayak [Phys. Rev. Lett. 77, 3009 (1996)].

The details are similar but too complex to discuss in a short time.

Repairing Nambu - Dyson by Eliashberg

$$\phi_{\ell'}(i\omega) = -\int_{-\infty}^{\infty} \frac{d\nu}{2\pi} V_{\ell'}(i\omega - i\nu) \times \frac{\phi_{\ell'}(i\nu)}{\sqrt{(\nu Z(i\nu))^2 + |\phi_{\ell'}(i\nu)|^2}}$$

$$[1 - Z(i\omega)]\omega = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} V_{\ell'}(i\omega - i\nu) \times \frac{\nu Z(i\nu)}{\sqrt{(\nu Z(i\nu))^2 + |\phi_{\ell'}(i\nu)|^2}}.$$

Although numerical details are a different but the qualitative answers are the same.

Z—factor and vertex correction

Calculations have been refined to include the Z-factor and the vertex correction. The previous results stand intact with little numerical effects

For these classes of problems the vertex correction can be shown to be zero from Ward identity. Thus Eliashberg equation is on firm grounds.

Kumar, Raghu and Mulligan, arXiv 1803.077767

Consider the Hall conductivity of composite fermions in the presence of quenched disorder using the non-relativistic theory of Halperin, Lee and Read (HLR). Consistent with the recent analysis of Wang et al., a fully quantum mechanical transport calculation, the HLR theory, under suitable assumptions, exhibits a Hall response $\sigma_{xy}^{CF} = -e^2/2h$ that is consistent with an emergent particle-hole symmetric electrical response. Remarkably, this response of the HLR theory remains robust even when the disorder range is of the order of the Fermi wavelength.

Nicholas Rombes and S. C. unpublished

Low temperature specific heat for the Son CF is

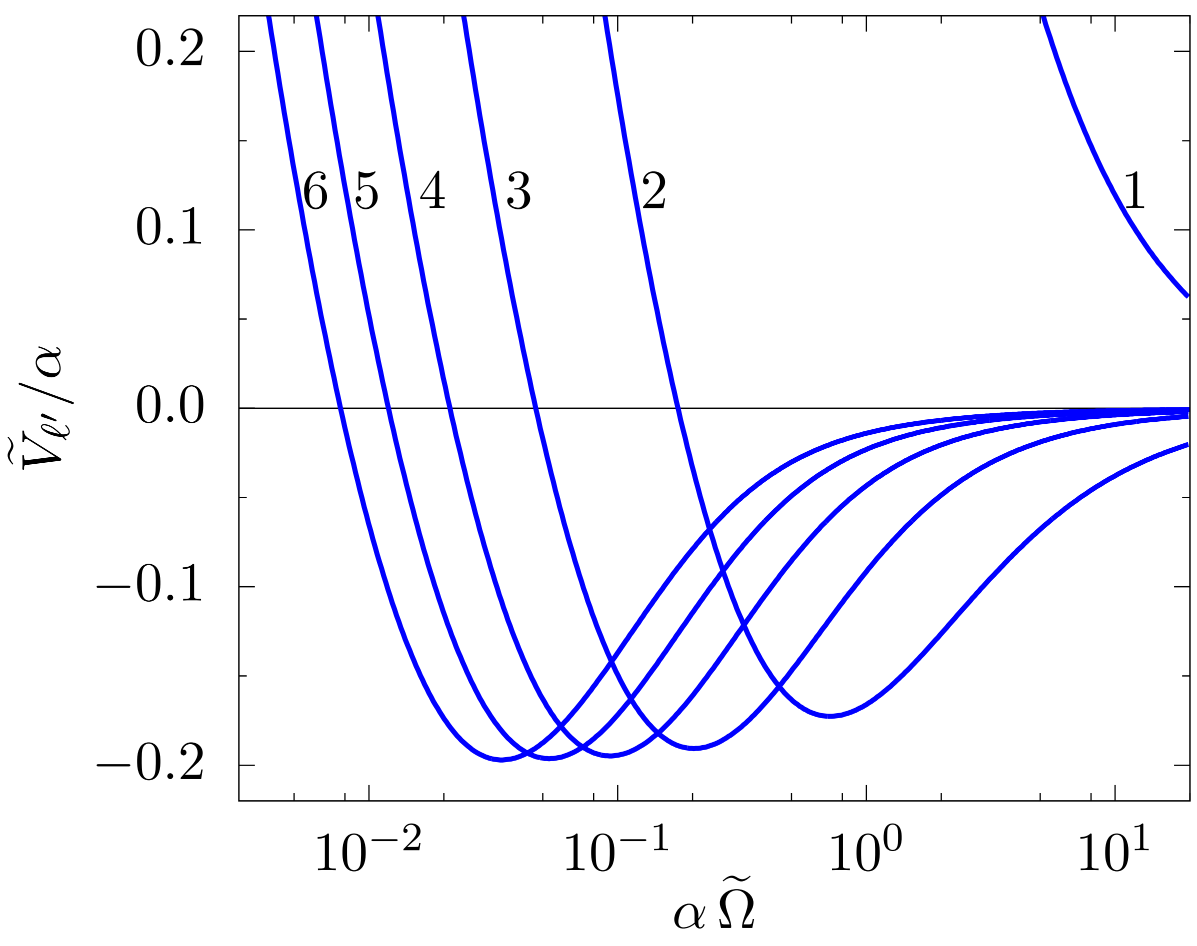
$$C \propto T \ln T$$

Same as in HLR theory

Tempting to speculate that Son CF theory is subsumed by HLR theory !!!

No particle-hole symmetric Pfaffian state as suggested by Son

Recall



$\ell = \ell' - 1 = 0$ **is repulsive for all frequencies**