Quantum simulation of the Polyakov loop for the Abelian Higgs model

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Outline:

Main point: proposal for cold-atom quantum simulator for the Abelian Higgs model in 1+1 dimensions.

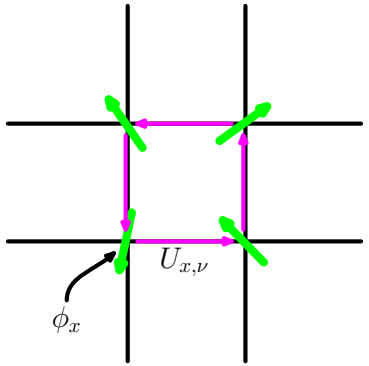
- Motivation
- The lattice Abelian Higgs model (scalar QED) in 1+1 dimensions (PRD 92, 076003)
- The Hamiltonian
- Data collapse for Polyakov loop (arXiv:1803.11166)
- Possible experimental realization: cold atoms on physical ladders + Rydberg-dressed interactions
- Simpler limits: quantum Ising model and the O(2) model
- Conclusions

Motivation

- Lattice QCD has been very successful at establishing that QCD is the theory of strong interaction, however some aspects remain inaccessible to classical computing.
- Finite density calculations: sign problem with Monte Carlo calculations with complex actions.
- Real time evolution: requires detailed information about the Hamiltonian and the eigenstates which is usually not available from conventional MC simulations at Euclidean time.
- Successes in using cold atoms to quantum simulate Condensed Matter systems (Mott transition, AFM order, BCS-BEC crossover, etc).

The Abelian Higgs model on a 1+1 space-time lattice

- complex (charged) scalar field $\phi_x = |\phi_x| e^{\imath\theta_x}$ attached to the space-time sites x
- Abelian gauge field $U_{x,\mu} = e^{iA_\mu(x)}$ on the links from x to $x + \hat{\mu}$.



$$S = S_g + S_h + S_\lambda, \qquad Z = \int D\phi^\dagger D\phi DU e^{-S}$$

where the gauge part is:

$$S_g = -\beta_{pl} \sum_x \sum_{\nu<\mu} \mathrm{ReTr}\left[U_{x,\mu\nu}\right]$$

$$U_{x,\mu\nu} = U_{x,\hat{s}} U_{x+\hat{s},\hat{\tau}} U^\dagger_{x+\hat{\tau},\hat{s}} U^\dagger_{x,\hat{\tau}} \to \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$$

The Abelian Higgs model on a 1+1 space-time lattice

the hopping part is:

$$
\begin{aligned}
S_h &= -\kappa_\tau \sum_x [e^{\mu} \phi_x^\dagger U_{x,\hat{\tau}} \phi_{x+\hat{\tau}} + e^{-\mu} \phi_{x+\hat{\tau}}^\dagger U_{x,\hat{\tau}}^\dagger \phi_x] \\
&\quad -\kappa_s \sum_x [\phi_x^\dagger U_{x,\hat{s}} \phi_{x+\hat{s}} + \phi_{x+\hat{s}}^\dagger U_{x,\hat{s}}^\dagger \phi_x] \rightarrow (D_\mu \phi)^\dagger D^\mu \phi
\end{aligned}
$$

and the self-interaction is:

$$
S_\lambda = \lambda \sum_x \left(\phi_x^\dagger \phi_x - 1 \right)^2 + \sum_x \phi_x^\dagger \phi_x
$$

In this talk $\lambda \rightarrow \infty$, $\phi_x^\dagger \phi_x$ is frozen to 1. The Abelian Higgs model becomes:

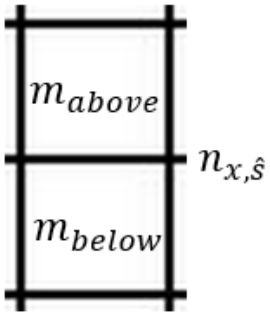
$$
\begin{aligned}
S &= -\beta_{pl} \sum_x \cos\left(A_{x,\hat{s}} + A_{x+\hat{s},\hat{\tau}} - A_{x+\hat{\tau},\hat{s}} - A_{x,\hat{\tau}}\right) \\
&\quad -2\kappa_\tau \cos\left(\theta_{x+\hat{\tau}} - \theta_x + A_{x,\hat{\tau}} - \imath\mu\right) - 2\kappa_s \cos\left(\theta_{x+\hat{s}} - \theta_x + A_{x,\hat{s}}\right)
\end{aligned}
$$

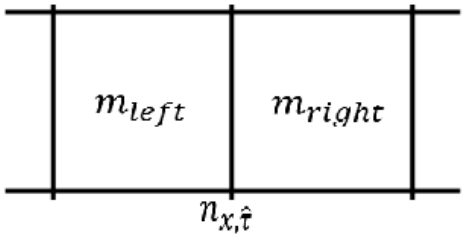
The $\lambda \to \infty$ limit: discrete formulation

The Fourier expansion
$exp[2\kappa_\nu cos(\theta_{x+\hat{\nu}} - \theta_x + A_{x,\hat{\nu}})] = \sum_{n=-\infty}^{\infty} I_n(2\kappa_\nu) exp(\imath n(\theta_{x+\hat{\nu}} - \theta_x + A_{x,\hat{\nu}}))$
leads to the partition function in terms of discrete sums:

$$e^{-S_{eff}} = \sum_{\{m_\square\}} \left\{ \prod_\square I_{m_\square}(\beta_{pl}) \prod_x \left[I_{n_{x,\hat{s}}}(2\kappa_s) \times I_{n_{x,\hat{\tau}}}(2\kappa_\tau) \exp(\mu n_{x,\hat{\tau}}) \right] \right\},$$

Where $n_{x,\hat{s}} = m_{below} - m_{above}$, $n_{x,\hat{\tau}} = m_{right} - m_{left}$ (Gauss's Law).





Attaching tensors to plaquettes and links

Define $t_n(z) \equiv I_n(z)/I_0(z)$ and attach:

- $B^{(\square)}$ tensor to every plaquette:

$$B^{(\square)}_{m_1 m_2 m_3 m_4} = \begin{cases} t_{m_\square}(\beta_{pl}), & \text{if } m_1 = m_2 = m_3 = m_4 = m_\square \\ 0, & \text{otherwise.} \end{cases}$$

- $A^{(s)}$ tensor to each horizontal link:

$$A^{(s)}_{m_{up} m_{down}} = t_{|m_{down} - m_{up}|}(2\kappa_s),$$

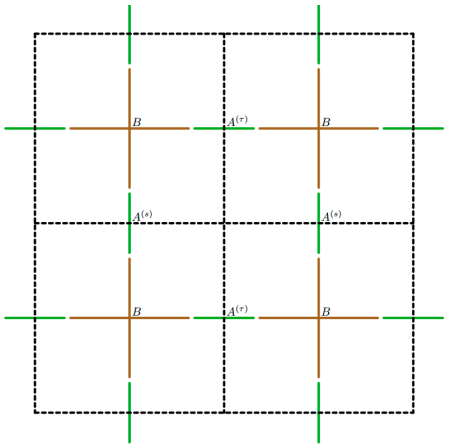
- $A^{(\tau)}$ tensor to each vertical link:

$$A^{(\tau)}_{m_{left} m_{right}} = t_{|m_{left} - m_{right}|}(2\kappa_\tau)\ \mathrm{e}^{\mu(m_{right} - m_{left})}.$$

Tensor Renormalization Group approach: $Z = Tr[\prod T]$

$$Z = \propto \mathrm{Tr}\left[\prod_{h,v,\square} A^{(s)}_{m_{up} m_{down}} A^{(\tau)}_{m_{right} m_{left}} B^{(\square)}_{m_1 m_2 m_3 m_4}\right]$$

The traces are performed by contracting the indices as shown:



The quantum numbers on the links are completely determined by the quantum numbers of the plaquettes.

$$Z = \propto \mathrm{Tr}\left[\mathbb{T}^{N_\tau}\right], \qquad \mathbb{T} = \sqrt{\mathbb{B}}\mathbb{A}\sqrt{\mathbb{B}}$$

The model

- This model has a continuous-time limit (always *gauge invariant*).
- The new variables have a *discrete* spectrum.
- Continuous-time limit: take $\beta_{pl}, \kappa_\tau \to \infty$, and $\kappa_s, a \to 0$, such that

$$U \equiv \frac{1}{\beta_{pl} a} = \frac{g^2}{a}, \quad Y \equiv \frac{1}{2\kappa_\tau a}, \quad X \equiv \frac{2\kappa_s}{a}$$

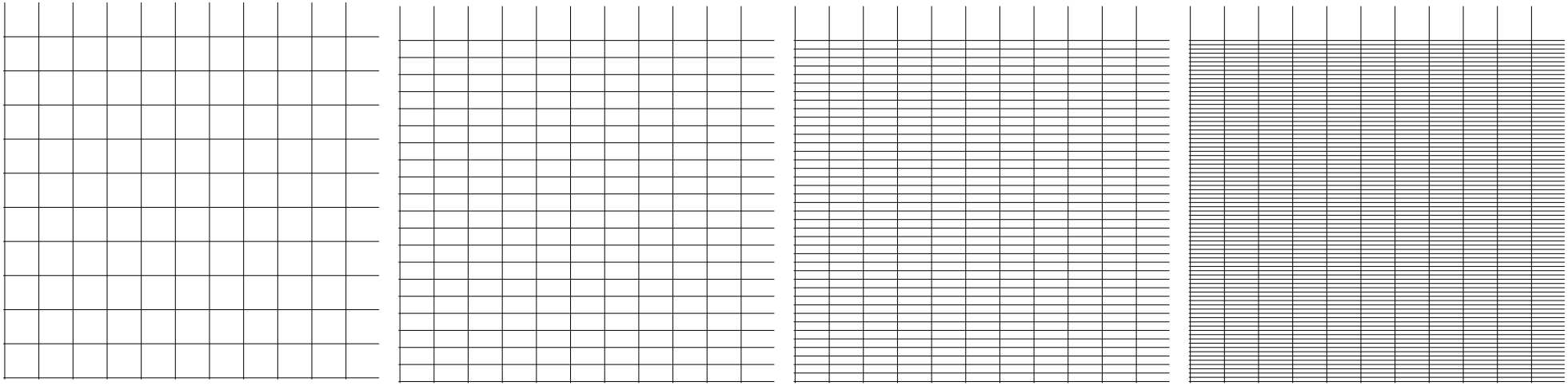
  are held constant.
- The Hamiltonian for $1 \ll \beta_{pl} \ll \kappa_\tau$ is:

$$H = \frac{U}{2}\sum_{i=1}^{N_s}(L_i^z)^2 + \frac{Y}{2}\sum_i{}'(L_{i+1}^z - L_i^z)^2 - X\sum_{i=1}^{N_s} U_i^x$$

  with

$$L^z|m\rangle = m|m\rangle, \quad U^x = \frac{1}{2}(U^+ + U^-), \quad U^\pm|m\rangle = |m \pm 1\rangle.$$

The time continuum limit of Abelian Higgs model



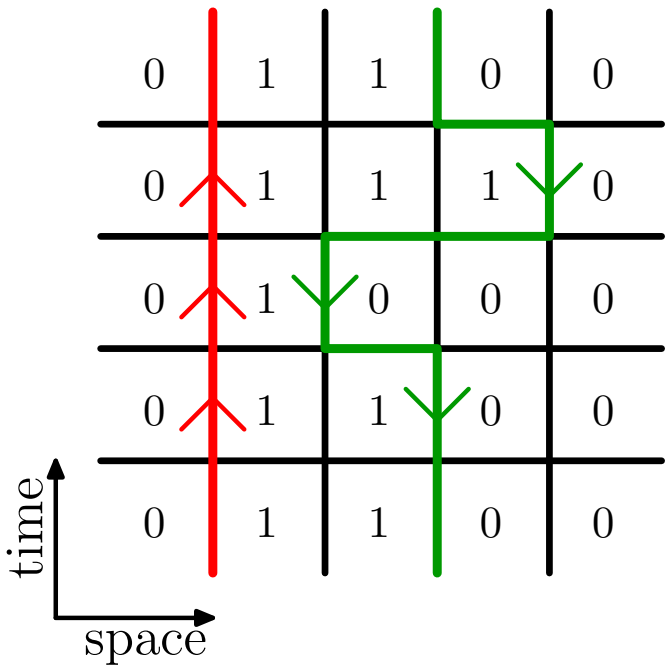
original lattice $\rightarrow$ | a, κ_s smaller & β_{pl}, κ_τ larger | a, κ_s smaller & β_{pl}, κ_τ larger | a, κ_s smaller & β_{pl}, κ_τ larger

The observable

Polyakov loop: a Wilson loop wrapped around the temporal direction of the lattice. This operator:

- is a product of gauge fields in the time direction.
- is an order parameter for confinement in gauge theories.
- has a continuous-time limit which adds a term to the Hamiltonian

$$P = \prod_{n=1}^{N_\tau} U_{x^*+n\hat{\tau},\tau}.$$



$$H \to H' = \frac{U}{2}\sum_{i=1}^{N_s}(L_i^z)^2 + \frac{Y}{2}\sum_{i\neq\frac{N_s}{2}}{}'(L_{i+1}^z - L_i^z)^2$$

$$+ \frac{Y}{2}(L_{\frac{N_s}{2}+1}^z - L_{\frac{N_s}{2}}^z - 1)^2 - X\sum_{i=1}^{N_s} U_i^x$$

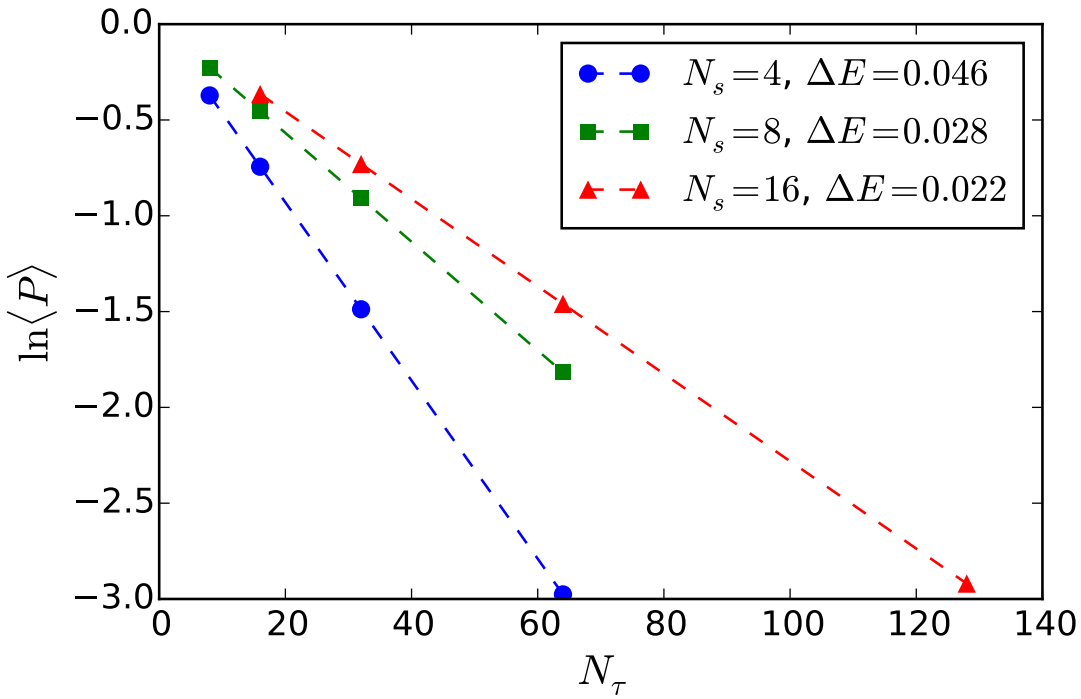
Polyakov loop: Lagrangian data

- Initial work led us to confirm that for large N_τ:

$$\langle P \rangle \simeq e^{-a\Delta E N_\tau}$$

with $aN_\tau = \frac{1}{T}$.

- ΔE is the energy gap between a system with a Polyakov loop, and one without.
- We investigated the finite-size scaling of ΔE and its dependence on $\beta_{pl} = 1/g^2$ and κ.



Polyakov loop collapse: Lagrangian data

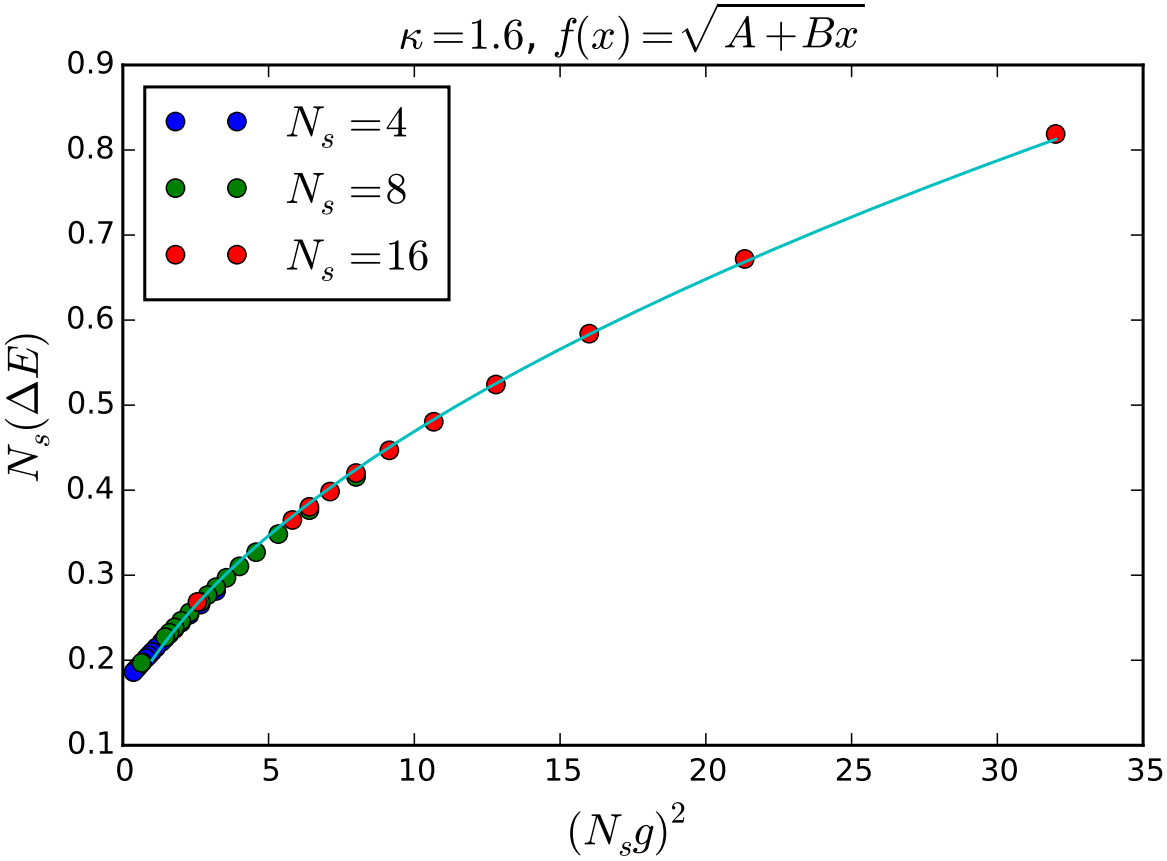
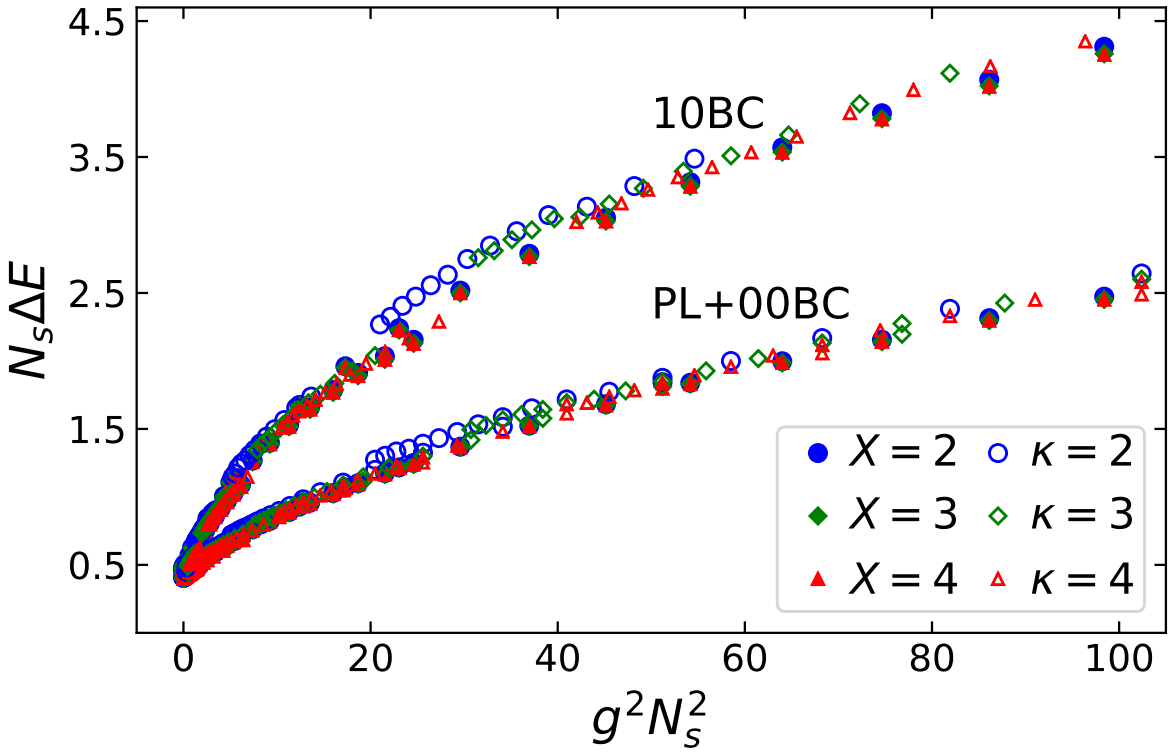
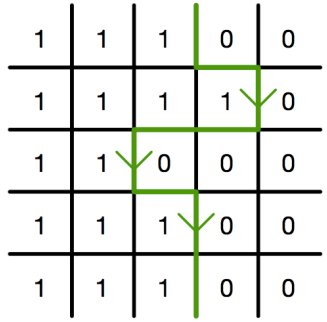
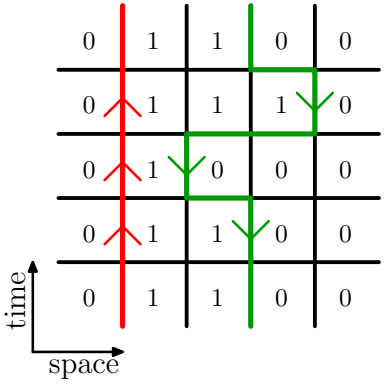


Figure: A fit to the universal curve of the form $\sqrt{A + Bx}$. In this calculation, space and Euclidean time are treated **isotropically**.

P-loop: universal function + collapse across limits

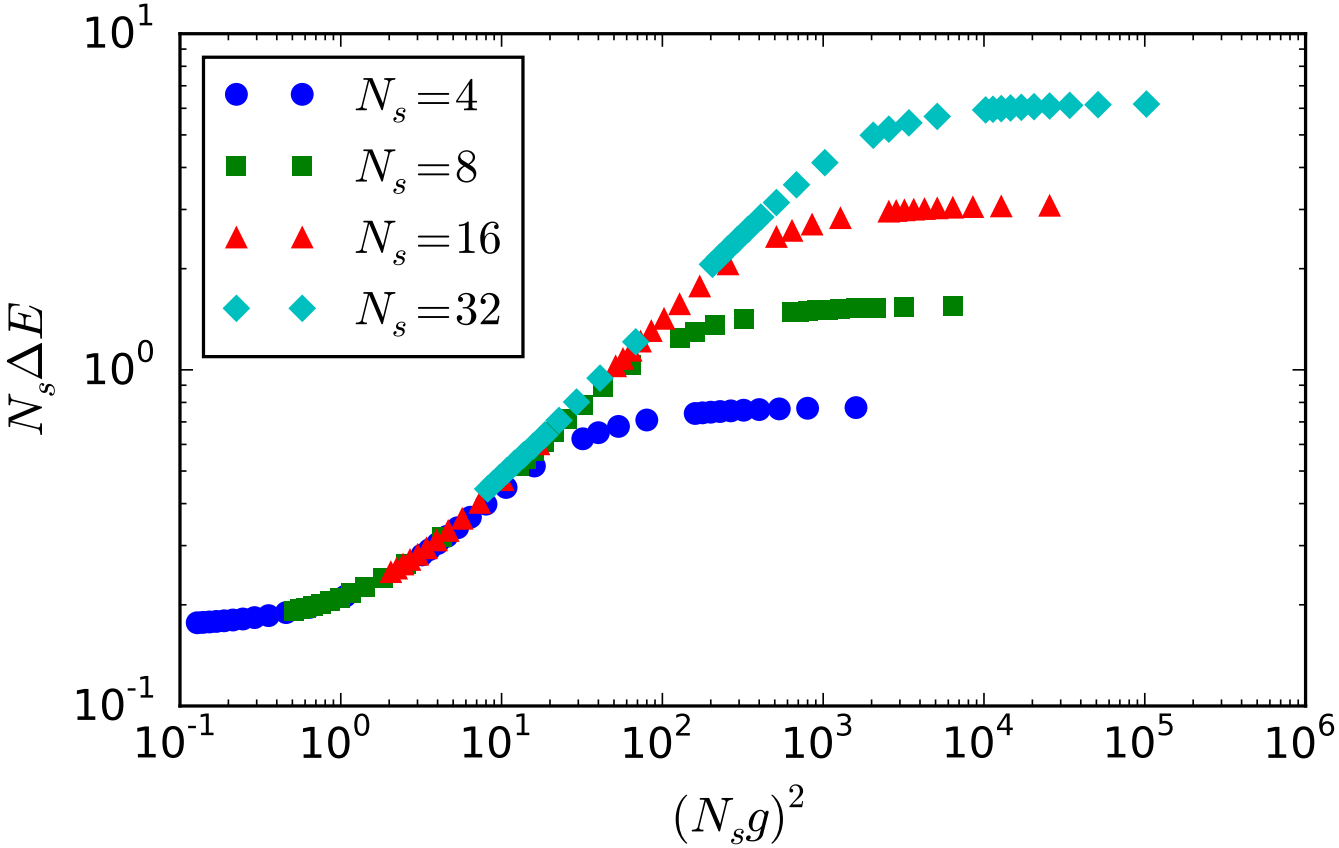






Data collapse of $N_s \Delta E$ defined from the insertion of the Polyakov loop (lower set) or with 1-0 boundary conditions (upper set), as a function of UN_s^2, or $g(N_s)^2$ (collapse of 24 data sets each).

P-loop collapse breaking: small N_s, large g (large U)

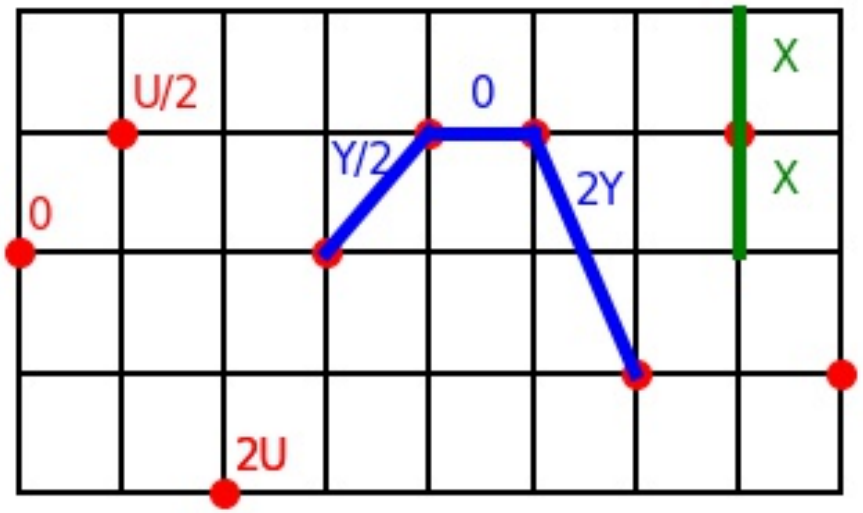


Data collapse across different N_s for sufficiently small g, and collapse breaking across different N_s at large g in the case of isotropic coupling. Here $\kappa = 1.6$, and $D_{\text{bond}} = 41$ was used in the HOTRG calculations.

Optical lattice implementation with a multi-leg ladder

$$\bar{H} = \frac{\tilde{U}_g}{2}\sum_i \left(\bar{L}^z_{(i)}\right)^2 + \frac{\tilde{Y}}{2}\sum_i (\bar{L}^z_{(i)} - \bar{L}^z_{(i+1)})^2 - \tilde{X}\sum_i \bar{L}^x_{(i)}$$

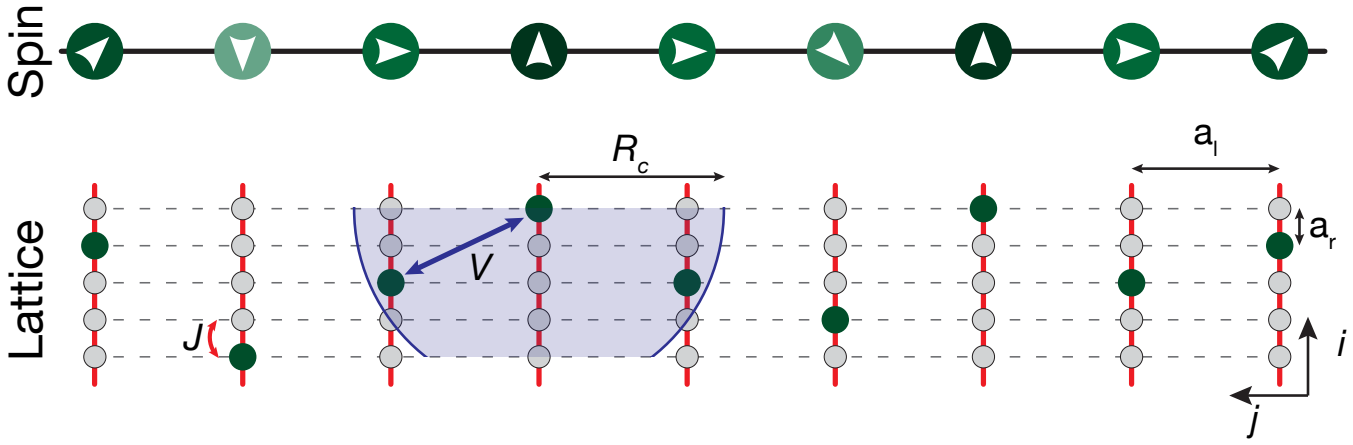
5 states ladder with 9 rungs



Ladder with one atom per rung: tunneling along the vertical direction, no tunneling in the horizontal direction, nearest-neighbor-rung attractive interaction. A parabolic potential is applied in the spin (vertical) direction.

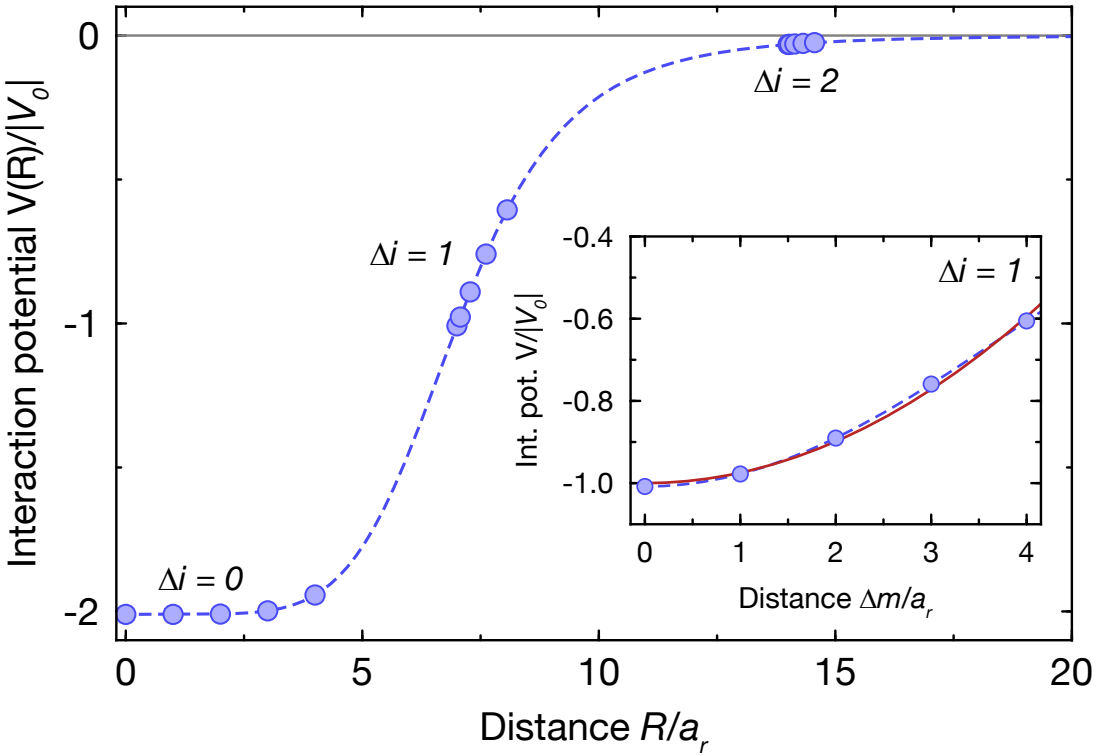
Experimental proposal (arXiv:1803.11166)

Cold atoms on multi-leg ladder with Rydberg-dressed interaction



$$\hat{H} = -\frac{J}{2}\sum_{i=1}^{N_s}\sum_{m=-s}^{s-1}\left(\hat{a}^{\dagger}_{m,i}\hat{a}_{m+1,i} + h.c\right) - \sum_{i=1}^{N_s}\sum_{m=-s}^{s}\epsilon_{m,i}\hat{n}_{m,i}$$
$$+\sum_{i,i'=1}^{N_s}\sum_{m,m'=-s}^{s} V_{m,m',i,i'}\,\hat{n}_{m,i}\hat{n}_{m',i'}$$

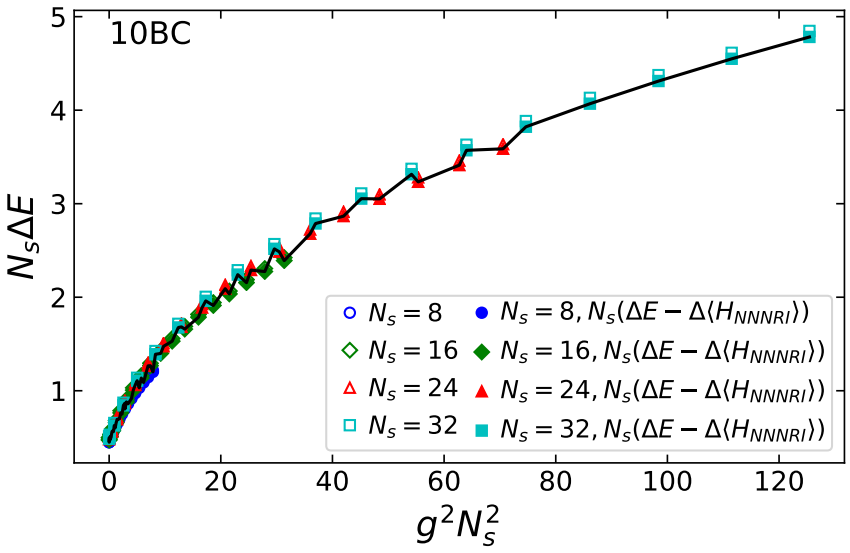
Rydberg-dressed potential

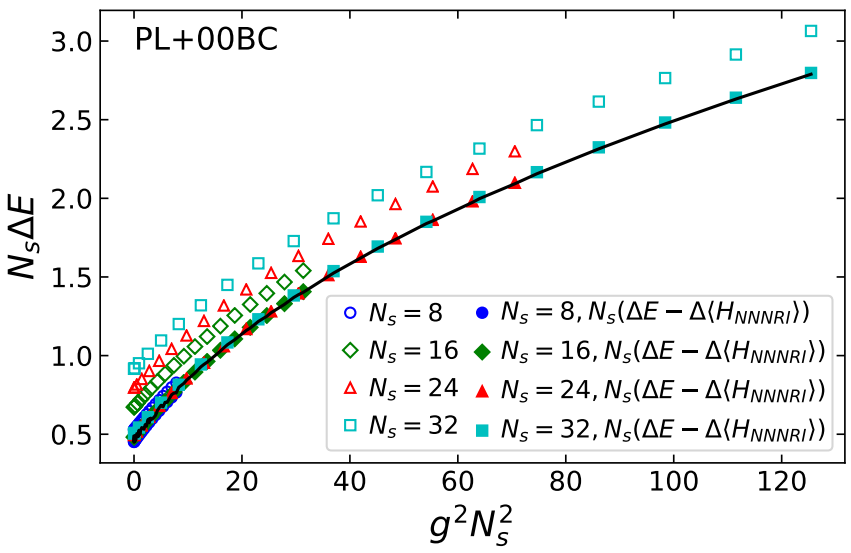


$$V(R) = \frac{U_0}{(1+(R/R_c)^6)}, \text{ with } R = \sqrt{(\Delta m\, a_r)^2 + (\Delta i\, a_l)^2}.$$

For this figure: $R_c = a_l = 7a_r$.

Effects of next-nearest-neighbor-rung interactions

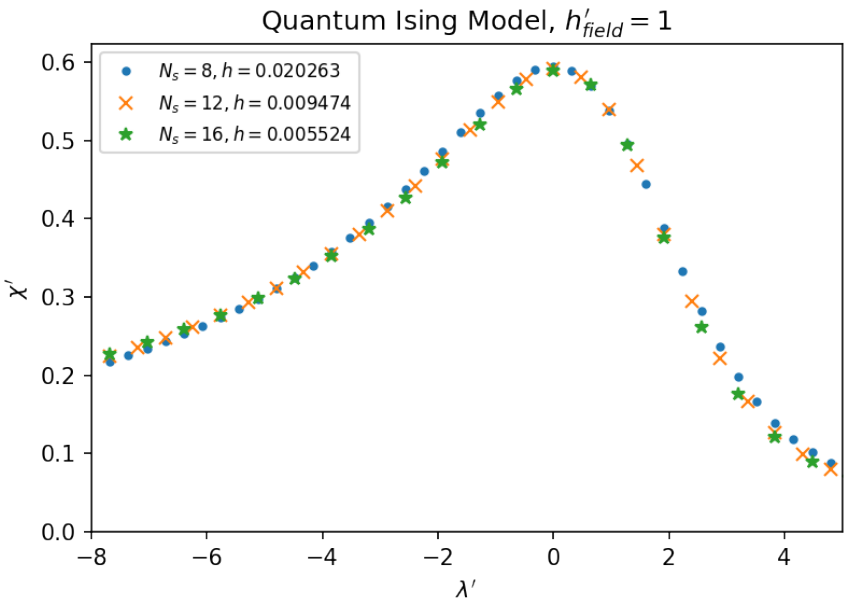




The quantum Ising model

In the case of 2 legs, we have the $S = 1/2$ quantum Ising model:

$$\hat{H} = -\lambda \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z - \sum_i \hat{\sigma}_i^x - h \sum_i \hat{\sigma}_i^z$$



$$\chi^{quant} = \frac{1}{N_s} \sum_{<i,j>} \langle (\sigma_i - \langle \sigma_i \rangle)(\sigma_j - \langle \sigma_j \rangle) \rangle \propto \xi^{1-\eta} \propto |\lambda - 1|^{-\nu(1-\eta)}$$

Data collapse: $\chi^{quant\prime} = \chi^{quant} N_s^{-(1-\eta)}$, $\lambda' = N_s^{1/\nu}(\lambda - 1)$, $h' = h N_s^{15/8}$

O(2) model and time continuum limit

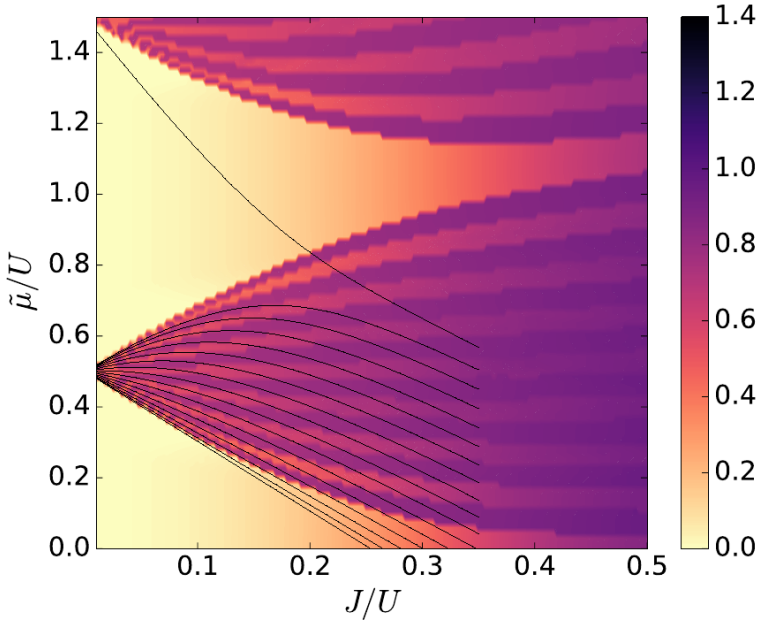
For $\beta_{pl} \to \infty, \lambda \to \infty$, we have the $O(2)$ model, and the Hamiltonian:

$$\hat{H} = \frac{\tilde{U}}{2} \sum_i \left(\hat{L}^z_{(i)}\right)^2 - \tilde{\mu} \sum_i \hat{L}^z_{(i)} - \frac{\tilde{J}}{4} \sum_i \left(\hat{L}^+_{(i)} \hat{L}^-_{(i+1)} + \hat{L}^-_{(i)} \hat{L}^+_{(i+1)}\right) ,$$

[$\hat{H}$ can also be obtained by following Fradkin, Susskind, Kogut, Polyakov, ...]

For large $\tilde{U} = \frac{1}{2\kappa_\tau a}$ and $\tilde{\mu} \simeq \frac{\tilde{U}}{2}$, the $O(2)$ model is approximated by a single-species Bose Hubbard model:

$$\hat{H}_{BH} = \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) - J \sum_i \left(\hat{a}^+_{(i)} \hat{a}^-_{(i+1)} + \hat{a}^-_{(i)} \hat{a}^+_{(i+1)}\right) ,$$



- $N_s = 16$ OBC
- Color is S_2 of time-continuum $O(2)$
- Stripes are jumps in particle number
- Black lines are particle number boundaries for Bose Hubbard

Entanglement entropy of O(2)

- n-th order Rényi entropy: $S_n(A) = \frac{1}{1-n}\ln(\mathrm{Tr}(\hat{\rho}_A^n))$.
- Calabrese-Cardy CFT predicts (to leading order):

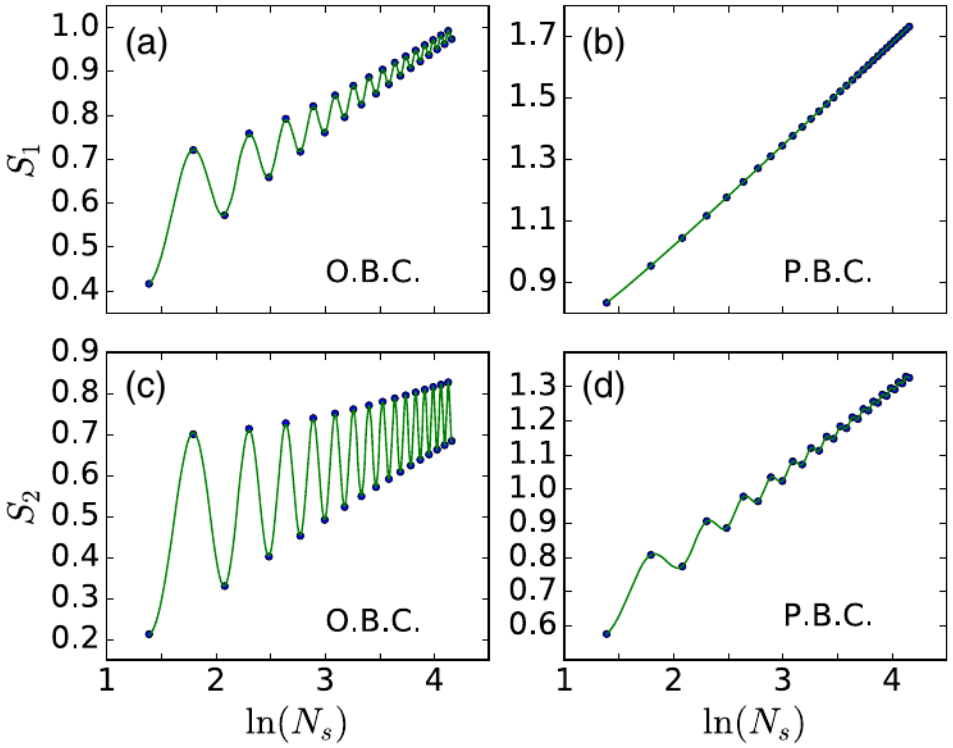
$$S_n(N_s) = \begin{cases} K_n + \frac{c(n+1)}{6n}\ln N_s & \text{for PBC} \\ K_n' + \frac{c(n+1)}{12n}\ln N_s & \text{for OBC.} \end{cases}$$

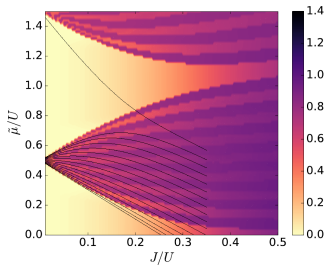
  where the central charge $c = 1$ for $O(2)$.

- Including all finite-size corrections that we know, the function to fit S_n data of small volumes is

$$S_n(N_s, l) = B_n + A_n \ln\left[N_s \sin\left(\frac{\pi l}{N_s}\right)\right] + C_n \frac{cos(\pi l)}{(N_s)^{p_n}}\left|sin\left(\frac{\pi l}{N_s}\right)\right|^{-p_n} + \frac{D_n}{\ln^2(N_s)}$$

Rényi entropy and fit coefficients for O(2)



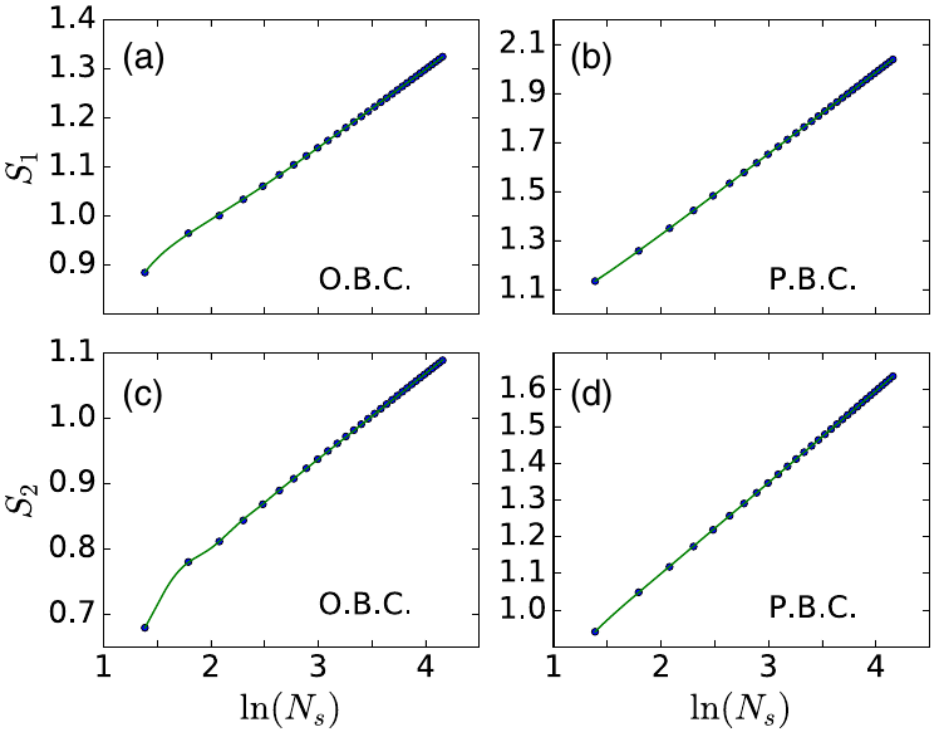


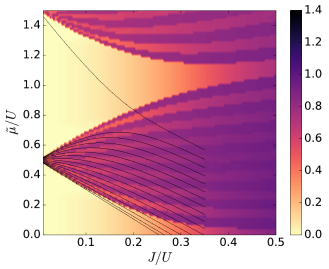
$$4\kappa_s\kappa_\tau = 0.01, 2\mu\kappa_\tau = 0.5$$

S_n	p_n	A_n from fit	A_n with $c = 1$
S_1 PBC	2.315	0.3338	$0.\bar{3}$
S_2 PBC	0.981	0.2525	0.25
S_1 OBC	0.901	0.1663	$0.1\bar{6}$
S_2 OBC	0.443	0.1246	0.125

Left: The 1st-order and 2nd-order Rényi entropies scaling with system size for $4\kappa_s\kappa_\tau(\tilde{J}/\tilde{U}) = 0.01, 2\mu\kappa_\tau(\tilde{\mu}/\tilde{U}) = 0.5$ in time continuum limit. **Right:** Values for A_n and p_n from the least-squares fits to DMRG data up to $N_s = 64$.

Rényi entropy and fit coefficient for O(2)



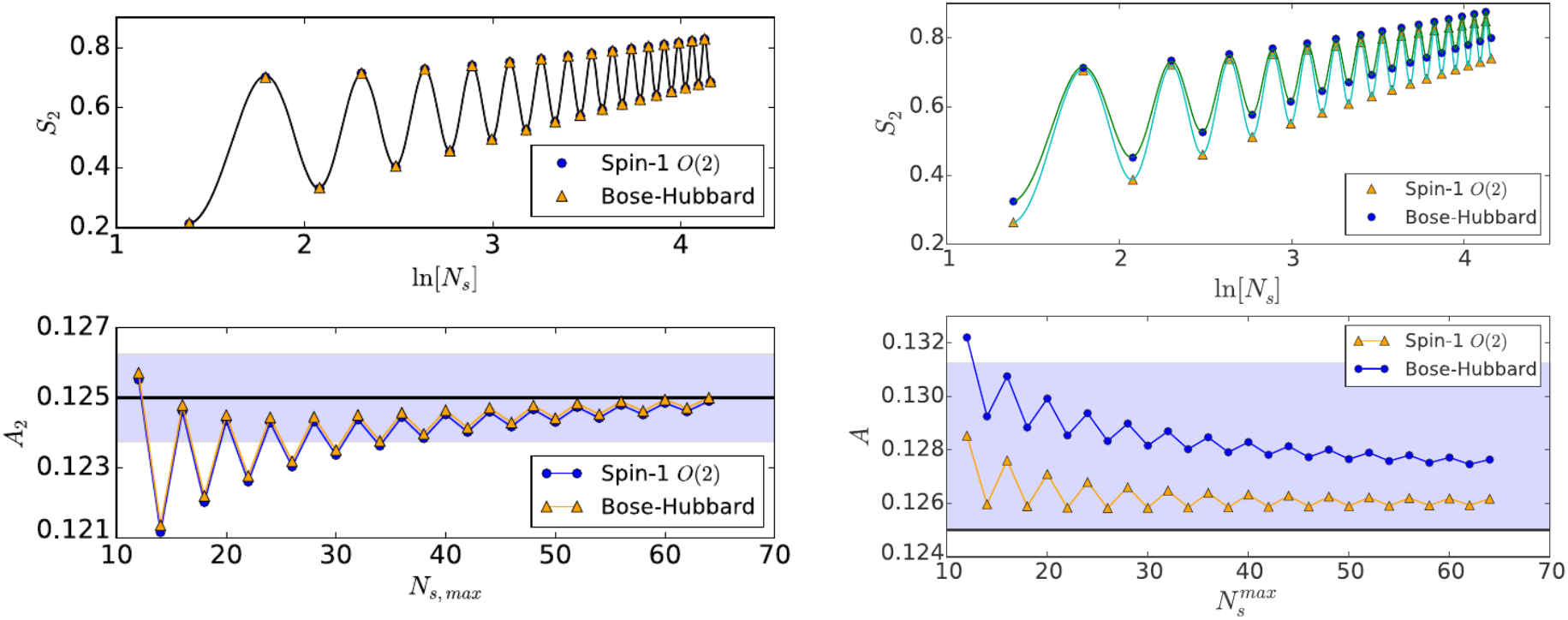


$$4\kappa_s\kappa_\tau = 4, 2\mu\kappa_\tau = 0$$

S_n	A_n from fit	A_n with $c = 1$
S_1 PBC	0.3337	$0.\bar{3}$
S_2 PBC	0.2500	0.25
S_1 OBC	0.1654	$0.1\bar{6}$
S_2 OBC	0.1278	0.125

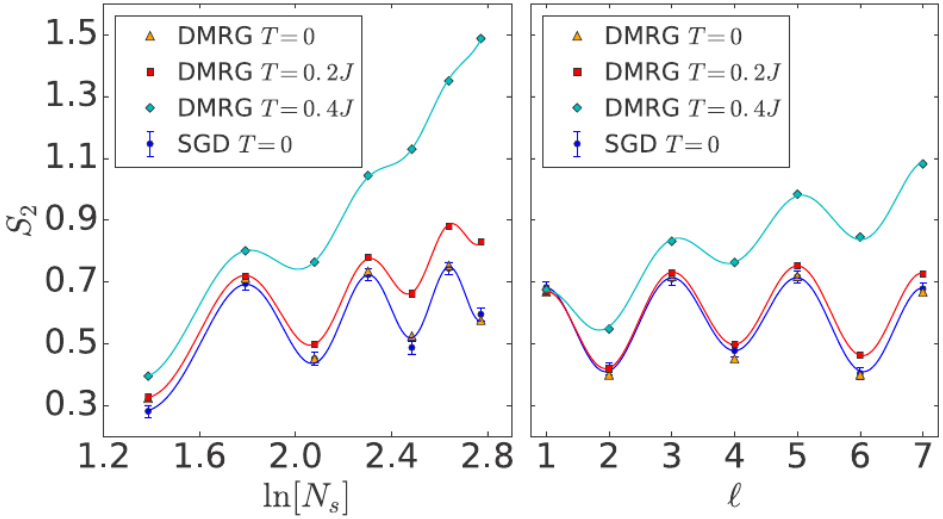
Left: The 1st-order and 2nd-order Rényi entropy scaling with system size for $4\kappa_s\kappa_\tau(\tilde{J}/\tilde{U}) = 4, 2\mu\kappa_\tau(\tilde{\mu}/\tilde{U}) = 0$ in time continuum limit. **Right:** Values for A_n from the least-squares fits to DMRG data up to $N_s = 64$.

Rényi entropy and fit coefficient for $O(2)$ and BH

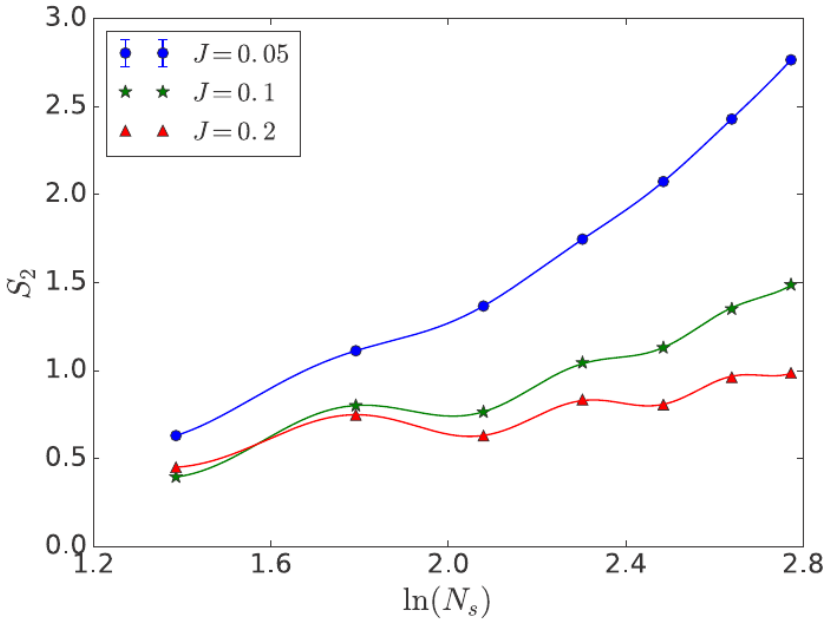


Top: S_2 at half-filling with OBCs for the Bose-Hubbard model and the $O(2)$, $J/U = 0.005$ (left) and $J/U = 0.1$ (right). **Bottom:** Values of A_2 as a function of the maximal values of N_s used in the fit, the bands represent departures of 1% (left) and 5% (right) from the expected value of 0.125.

Finite Temperature Effects



S_2 at half-filling for BH systems with $J/U = 0.1$ and synthetically generated data (SGD) with random Gaussian fluctuations with $\sigma_{S_2} = 0.02$. We find $\sigma_{A_2} \simeq 3.1\sigma_{S_2}$.



S_2 as a function of the logarithm of system size with a finite temperature and $l = N_s/2$. Here $U = 1$ and $T = 0.04$.

Conclusions

- We have proposed a gauge-invariant approach for the quantum simulation of the Abelian Higgs model.
- The tensor renormalization group approach provides a discrete formulation in the limit $\lambda \to \infty$ (suitable for quantum computing)
- Calculations of the Polyakov loop at finite N_s and small gauge coupling show a universal behavior.
- A ladder of cold atoms with N_s rungs, one atom per rung, and $2s + 1$ legs is a candidate system for experimental realization
- Proof of principle: data collapse for the quantum Ising model.
- The O(2) limit can be simulated with simple Bose-Hubbard model to extract universal quantities.

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Collaborators and group

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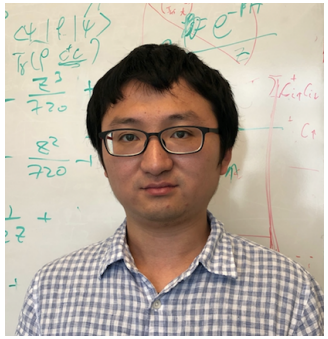

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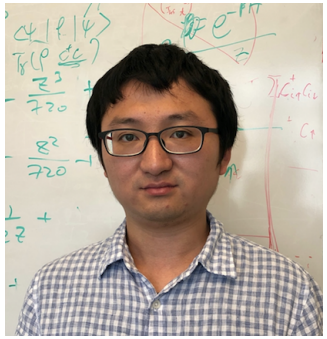

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